# Multiple-Hypothesis and Graph-Based Approaches to Multi-Target Tracking


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## *ABSTRACT*

*This manuscript discusses aspects of association-based multi-target tracking, with a particular focus on the multiple-hypothesis tracking (MHT) paradigm. We address two recent extensions for challenging multi-sensor tracking problems with disparate sensors. One approach considers a novel asynchronous processing framework for MHT. The second approach relies on simplifying approximations to enable scalable graph-based processing. We describe initial simulation-based results, and discuss directions for further research.*


## 1.0 MULTI-TARGET TRACKING

In multi-target tracking (MTT), the variable of interest over a sequence of times $t^k = (t_1, \ldots, t_k)$ is a set of trajectories that we denote by $X^k$. Each trajectory in this set has a time of birth, an evolution in target state space, and (possibly) a time of death. Hence, we are interest to identify the time evolution of an unknown (and time-varying) number of objects. We observe a sequence of sets of measurements $Z^k$. The usual simplifying assumption in the MTT problem formulation is that each target at each sensor measurement time gives rise to at most one measurement. However, it is not known which measurement originates from which object, and there are as well false measurements that are not target originated.

### 1.1 Optimality

A first difficulty in addressing the MTT problem is that it is not obvious how to define optimality. One could consider the posterior probability distribution $p(X^k|Z^k)$ and seek to identify a classical estimator, e.g. the *maximum a posteriori* (MAP) estimator. Aside from the computational complexity associated with attempting such an operation, there is a conceptual difficulty in performing MAP estimation in this setting. This issue is discussed effectively by Mahler in [1, pp. 494-500]. Essentially, it is problematic to compare values of the posterior probability distribution for choices of $X^k$ that correspond to sets of objects with disparate cardinalities or temporal support. MAP estimation cannot meaningfully be performed.

One approach to resolving this difficulty is explicitly to consider an explanation for the data, i.e. to specify which measurements are to be rejected as false and how target-originated measurements are to be associated. Let us denote by $q^k$ one such global hypothesis or explanation. This leads to a probabilistic conditioning approach and the following expression for the multi-target posterior probability distribution $p(X^k|Z^k)$:

$$p(X^k|Z^k) = \sum_{q^k} p(X^k|Z^k, q^k) p(q^k|Z^k). \tag{1}$$

It is worth noting that the space of global hypotheses is enormous. (In fact, considering the possibility that not all targets will be detected over the time interval $t^k$, the set of global hypotheses is infinite.) The *multiple-hypothesis tracking* (MHT) paradigm resolves the conceptual and practical difficulties noted above. The former difficulty is addressed by focusing exclusively on $p(q^k|Z^k)$, and seeking the MAP estimate for $q^k$ without facing the conceptual difficulty posed by continuous-valued spaces. The latter difficulty is generally addressed by neglecting undetected targets, and resolving hypotheses over small time horizons to bound computational complexity.

Thus, the MHT approach may be characterized as seeking the best explanation of the data, and then conditioning on this explanation to determine the continuous-space trajectories of interest. The latter task amounts to solving a set of nonlinear filtering problems, for which in the linear Gaussian case both MMSE and MAP estimators are given by the recursive Kalman filter; in the linear non-Gaussian case, the Kalman filter remains optimal among all linear estimators:

$$\hat{q}^k = \arg\max_{q^k} p(q^k|Z^k), \tag{2}$$

$$\hat{X}^k = \arg\max_{X^k}\, p(X^k|Z^k, \hat{q}^k). \tag{3}$$

Association-based approaches like MHT do not directly optimize a criterion based on the multi-target posterior probability distribution $p(X^k|Z^k)$. Thus, while MHT seeks the MAP association hypothesis $\hat{q}^k$ – and this surely is a reasonable thing to do – there is no guarantee that selecting $\hat{q}^k$ will lead to optimal performance with respect to arbitrary MTT performance criteria based on $p(X^k|Z^k)$. An interesting result in this respect is discussed in [2], where communication-constrained data association is considered. It is shown that *statistical nearest neighbor* (SNN) association, i.e. the MAP solution to the single-target single-scan track maintenance problem, does not minimize the track localization error in sufficiently-high clutter environments. One can do better by following the data-association strategy detailed in the paper. Nonetheless, in practice (and not surprisingly) we find that selection of the MAP association hypothesis is generally a good thing to do.

## 1.2 The Use of Bayes Rule

In [3, pp. 8-9], Vo presents a lucid discussion of potential conceptual difficulties in association-based MTT. His focus is on the problematics associated with applying Bayes rule to manipulate $p(q^k|Z^k)$. In particular, Bayes rule prescribes the following:

$$p(q^k|Z^k) = \frac{p(Z^k|q^k)p(q^k)}{p(Z^k)}. \tag{4}$$

Vo rightly observes that the use of Bayes rule in this setting, while seemingly benign, does raise some concerns. Since $q^k$ depends on $Z^k$ as it prescribes how to explain the data, is $p(q^k)$ a valid prior? Likewise, is $p(Z^k|q^k)$ a valid likelihood function?

The use of Bayes rule in this setting can be clarified via a conditioning argument. To our knowledge, this point has been overlooked in the MHT community to date. In particular, we must proceed as follows, where $|Z^k|$ is the sequence of measurement set cardinality for the time sequence $t^k$:

$$p(q^k|Z^k) = p(q^k|Z^k, |Z^k|) = \frac{p(Z^k|q^k,|Z^k|)p(q^k||Z^k|)}{p(Z^k||Z^k|)}. \tag{5}$$

Note that $q^k$ is conditionally independent of $Z^k$ given $|Z^k|$, hence $p(q^k||Z^k|)$ is now a valid prior and $p(Z^k|q^k,|Z^k|)$ is now a valid likelihood function.

Referring to the numerator in (5), Vo notes further that it is not clear whether $p(Z^k|q^k)p(q^k)$ is the joint density $p(Z^k,q^k)$. Indeed, the marginal $p(q^k)$ that result by integration of $p(Z^k,q^k)$ over $Z^k$ according to (7) must not depend on the data, contracting the fact that $q^k$ does depend on $Z^k$. In (7), we denote by $Z$ the dummy integration variable. Note that (7) neglects the fact that the integration can only meaningfully be performed over those measurement sets that are consistent with $q^k$:

$$p(q^k) = \int_Z \; p(Z|q^k)p(q^k)dZ. \tag{6}$$

Once more, we can resolve this difficulty by considering instead the joint density conditioned on a given $|Z^k|$. Referring to the numerator in (5), we have:

$$p(q^k||Z^k|) = \int_{Z,|Z|=|Z^k|} p(Z|q^k,|Z^k|)p(q^k||Z^k|)dZ \tag{7}$$

Finally, the normalizing constant must again be understood as integration over all global hypotheses consistent with a given measurement cardinality. This resolves the concern raised by Vo as to whether the normalizing constant even exists. Note that $q^k$ is discrete-valued, hence (8) is simply a sum:

$$p(Z^k||Z^k|) = \int_{q^k \text{ consistent with } |Z^k|} p(Z^k|q^k,|Z^k|)p(q^k||Z^k|)dq^k. \tag{8}$$

## 1.3 Hypothesis-Oriented MHT

Computational and real-time constraints require that we adopt a recursive formulation of (5). Thus, we proceed as follows:

$$p(q^k|Z^k) = \frac{p(q^k|Z^k)}{p(|Z_k||Z^k)} = p(q^k|Z^k,|Z_k|) = \frac{p(Z_k|Z^{k-1},q^k,|Z_k|)p(q^k|Z^{k-1},|Z_k|)}{p(Z_k|Z^{k-1},|Z_k|)}. \tag{9}$$

We consider in turn each of the factors in (10). Noting that $|Z_k|$ is known given $q^k$, the first numerator factor in (9) may be manipulated as follows:

$$\begin{aligned} p(Z_k|Z^{k-1},q^k,|Z_k|) &= p(Z_k|Z^{k-1},q^k,|Z_k|)p(|Z_k||Z^{k-1},q^k) \\ &= p(Z_k,|Z_k||Z^{k-1},q^k) = p(Z_k|Z^{k-1},q^k). \end{aligned} \tag{10}$$

The second numerator factor in (9) may be manipulated as follows:

$$\begin{aligned} p(q^k|Z^{k-1},|Z_k|) &= p(q_k|Z^{k-1},|Z_k|,q^{k-1})p(q^{k-1}|Z^{k-1},|Z_k|) \\ &= \frac{p(q_k|Z^{k-1},|Z_k|,q^{k-1})p(|Z_k||q^{k-1},Z^{k-1})p(q^{k-1}|Z^{k-1})}{p(|Z_k||Z^{k-1})}. \end{aligned} \tag{11}$$

The denominator in (9) may be manipulated as follows:

$$p\left(Z_k|Z^{k-1}, |Z_k|\right) = \frac{p\left(Z_k|Z^{k-1}\right)}{p\left(|Z_k||Z^{k-1}\right)}. \tag{12}$$

Combining (11-13) according to (10) yields the following:

$$p\left(q^k|Z^k\right) = \frac{p\left(Z_k|Z^{k-1}, q^k\right)p\left(q_k|Z^{k-1}, q^{k-1}\right)p\left(q^{k-1}|Z^{k-1}\right)}{p\left(Z_k|Z^{k-1}\right)}. \tag{13}$$

This is the global hypothesis recursion that expresses $p\left(q^k|Z^k\right)$ as a function of $p\left(q^{k-1}|Z^{k-1}\right)$ and the current scan of data $Z_k$. This recursion matches what has already been presented in the literature, while addressing the conceptual concerns discussed previously. Note that $p\left(q_k|Z^{k-1}, q^{k-1}\right)$ in (13) may be simplified further to $p\left(q_k|q^{k-1}\right)$.

## 1.4 Track-Oriented MHT

Though useful, the recursion (13) is generally intractable in the sense that the space of global hypotheses is quite large. Fortunately, under some simplifying assumptions, namely Poisson-distributed number of target births and number of false alarms at each scan, the posterior probability of a global hypothesis $p\left(q^k|Z^k\right)$ may be expressed as a product over local (or *track*) hypotheses associated with $q^k$. This fundamental contribution to the MHT literature is presented in [4].

The Poisson assumptions above are quite reasonable in many settings. Indeed, consider a continuous-time birth-death process with exponentially-distributed target inter-arrival (birth) times with parameter $\lambda_b$, and exponentially distributed target lifetime with parameter $\lambda_\chi$. Discrete-time statistics may be readily obtained, leading to a Poisson distributed number of births with mean $\mu_b(t)$ and death probability $p_\chi(t)$ over an interval of duration $t$. The expressions are given in equations (14-15):

$$\mu_b(t) = \frac{\lambda_b}{\lambda_\chi}\left(1 - e^{-\lambda_\chi t}\right), \tag{14}$$

$$p_\chi(t) = 1 - e^{-\lambda_\chi t}\ . \tag{15}$$

For simplicity, in the following we will omit the time interval $t$ and use the birth rate and death probability $\mu_b$ and $p_\chi$, respectively. (Time arguments should be noted explicitly when the sensor revisit interval is time-varying.)

Similarly, the Poisson false alarm assumption (with mean $\Lambda$) is a reasonable one as it matches clutter statistics in many application domains. It results as a limiting case of the Binomial distribution with a large number of detection cells $N$ and vanishingly small false detection probability $p_F$, with $p_F \cdot N \to \Lambda$. We assume that at every scan, each target is detected with probability $p_d$.

Let $\tau$ be the number of targets in global hypothesis $q^{k-1}$ at time $t^{k-1}$, $r = |Z_k|$ be the number of measurements in the current scan at time $t^k$, and $b$, $\chi$, and $d$ are the number of target births, deaths, and measurement updates in global hypothesis $q^k$ at time $t^k$, respectively. Note that the classical MHT only considers global hypotheses for which targets are detected at birth.

We now express the global hypothesis recursion (13) explicitly. It can be shown that the factor $p\left(q_k|q^{k-1}\right)$ may be written as follows:

$$p\left(q_k|q^{k-1}\right) = \left\{\frac{\exp(-p_d\mu_b-\Lambda)\Lambda^r}{r!}\right\} p_\chi^\chi \left(\left(1-p_\chi\right)(1-p_d)\right)^{\tau-\chi-d} \left(\frac{(1-p_\chi)p_d}{\Lambda}\right)^d \left(\frac{p_d\mu_b}{\Lambda}\right)^b. \quad (16)$$

The factor $p\left(Z_k|Z^{k-1},q^k\right)$ in (13) accounts for the probability of observing a set of measurements given a global hypothesis. It is simply a product over filter residual scores; hence, it may be written as follows, where, under $q_k$, $J_d$ is the set of track update measurements, $J_{fa}$ is the set of false alarms, $J_b$ is the set of target birth measurements, and $f(\cdot)$ is the filter score:

$$p\left(Z_k|Z^{k-1},q^k\right) = \prod_{j\in J_d} f\left(z_j|Z^{k-1},q^k\right) \prod_{j\in J_b} f\left(z_j|Z^{k-1},q^k\right) \prod_{j\in J_{fa}} f\left(z_j|Z^{k-1},q^k\right). \quad (17)$$

Equations (16-17) may be combined into (13), resulting in the following track-oriented MHT recursion. Equation (18) is of fundamental importance in that it factors global hypothesis scores into track scores. This allows the recursive determination of $\hat{q}^k$ as the solution to an integer programming problem, without requiring explicit enumeration of global hypotheses. Note that $f_{fa}(\cdot)$ is the false alarm distribution:

$$p\left(q^k|Z^k\right) = p_\chi^\chi \left(\left(1-p_\chi\right)(1-p_d)\right)^{\tau-\chi-d} \prod_{j\in J_d} \frac{\left(1-p_\chi\right)p_d f\left(z_j|Z^{k-1},q^k\right)}{\Lambda f_{fa}\left(z_j\right)} \prod_{j\in J_b} \frac{p_d\mu_b f\left(z_j|Z^{k-1},q^k\right)}{\Lambda f_{fa}\left(z_j\right)}$$

$$\cdot \frac{\left\{\frac{\exp(-p_d\mu_b-\Lambda)\Lambda^r}{r!}\right\}\prod_{j\in Z_k} f\left(z_j|Z^{k-1},q^k\right)}{p\left(Z_k|Z^{k-1}\right)} p\left(q^{k-1}|Z^{k-1}\right). \quad (18)$$

## 1.5 Practical Considerations and Algorithmic Extensions

Though useful, the recursion (18) still is insufficient for viable MHT processing. Indeed, in principle one must form all track hypotheses over a temporal batch of data followed by solution to an optimization problem that results in $\hat{q}^k$. This incurs computational expense and solution latency in large surveillance problems.

Hypothesis pruning allows tractable computational expense. Effective pruning schemes exist, based on reduction to a single global hypothesis with a bounded temporal delay; this enables both reduced computations and real-time processing. A straightforward solution to the integer programming problem is via *linear programming* (LP) relaxation; this was studied independently in [5-6].

Optimal processing in principle requires full hypothesis formation (with no hypothesis pruning) as well as track extraction as a single processing step. As mentioned above, hypothesis pruning is necessarily required. Further, track extraction is generally performed only for resolved hypotheses. Thus, hypothesis resolution and track management are generally decoupled in most MHT implementations. Correspondingly, hypothesis resolution based on LP relaxation employs equality constraints; that is, all sensor measurements are accounted for in all global hypotheses [5].

While suboptimal, the use of distinct hypothesis resolution and track extraction functions offers processing advantages. In particular, confirmed tracks may be favored in data-association processing; see [7-8] for an analysis of advantages resulting from feedback processing from track extraction to data association functions.

Generally, we do not distinguish between the data-association hypothesis $\hat{q}^k$ and global hypotheses that are consistent with it. In general there any many data-indistinguishable global hypotheses associated with the same data association hypothesis. In particular, multiple target birth and death times are possible, and there may as well be targets with no associated detections. The merits of considering a larger hypothesis space and the ability to do so without incurring additional computation expense are discussed in [9].

Hypothesis aggregation for data-indistinguishable hypotheses takes at least two forms. One involves aggregating over all target birth and death times to result in a single (aggregated) track hypothesis for a single associated-measurement sequence [10]. The other involves aggregation over indistinguishable sensor measurements, as will occur in cardinality-estimation applications [11]. Aggregation over similar (but not data-indistinguishable) hypotheses may also be performed with appreciable benefits [12].

While centralized fusion provides excellent performance in many settings, effective exploitation of multi-sensor data with good performance and robustness characteristics often requires advanced processing architectures [13]. Fading detection statistics and sensor registration errors are best handled in distributed architectures. In addition to improved robustness characteristics, multi-stage data association provides an effective means to handling disparate sensor update rates and to exploit same-sensor association performance [14-15].

While the success of distributed processing solutions over centralized processing may surprise those familiar with detection and estimation theory and the optimality results associated with centralized solutions, we must recognize that the MTT problem is exceedingly complex. Hence, the choice is between suboptimal centralized solutions and suboptimal distributed solutions. Hence, distributed processing must be viewed as a flexible approach to suboptimal but effective surveillance solutions. Similarly, surprising results have been shown recently regarding the value of asynchronous processing in forensic settings to content with disparate data sources where the low-rate sensor is highly informative [16]. In such settings, the purposeful use of out-of-sequence processing enables effective MHT solutions that are impossible to achieve in time-sequential processing. This is described at greater length Section 2.

MTT with redundant measurements poses a significant challenge. For simplicity, most paradigms adapt a Bernoulli measurement model. There are some exceptions, e.g. the *probabilistic MHT* (PMHT) and its *non-generative* sensor model [17]. A complementary difficulty – merged measurements due to more than one target – also is not considered in most MTT treatments.

Redundant measurements induced by multipath phenomena or multiple emissions have been addressed in an MHT setting; see [18-19] and references therein. However, while these papers are of interest, they do not address the challenging problem where all redundant measurements are characterized by the *same* measurement equation. A recent treatment of redundant measurement in the context of *probability hypothesis density* (PHD) research is discussed in [20-22]. Both merged and redundant measurements are addressed using a *Markov Chain Monte Carlo* (MCMC) approach in [23], and in [24-25] with the *probabilistic data association filter* (PDAF). The generalized MHT approach for this problem is developed in [26].

## 1.6 Further Observations

We now clarify two additional points regarding the nature of the MHT solution to the MTT problem. First, the *track-oriented MHT* (TOMHT) solution as described here is an efficient means to achieving the *hypothesis-oriented MHT* (HOMHT) solution. Indeed, the two solutions will be the same under the assumption of Poisson-distributed target births and clutter returns. Hence, both approaches include MAP estimation of a global hypothesis, followed by *minimum mean squared error* (MMSE) filtering conditioned on the MAP solution to the data association problem. Thus, we may characterize both HOMHT and TOMHT as "MMSE-MAP" approaches. This observation is in contrast to some assertions in the literature that characterize TOMHT as a MMSE-ML approach [13, 27] that relies on a *maximum likelihood* (ML) criterion. In contrast to this claim, it should be noted that our

TOMHT relies on a *non-uniform a priori* distribution on target births. Nonetheless, it certainly might be true that depending on implementation details, other TOMHT implementations may be best characterized as MMSE-ML algorithms.

Second, Mahler in [1, pp. 340-341] raises the concern that the global hypothesis appears to be anomalous as a state variable. First, he asserts that since $|Z_k|$ is an observable, its use in the MHT recursion is suspicious. Let us examine this by looking again at (13), written here as (19) in slightly-simplified form:

$$p\left(q^k|Z^k\right) = \frac{p\left(Z_k|Z^{k-1},q^k\right)p\left(q_k|q^{k-1}\right)p\left(q^{k-1}|Z^{k-1}\right)}{p\left(Z_k|Z^{k-1}\right)}. \tag{19}$$

Note that we may understand this as a prediction-update recursion, whereby first $p\left(q^{k-1}|Z^{k-1}\right)$ is predicted by use of $p\left(q_k|q^{k-1}\right)$. That is, we have:

$$p\left(q^k|Z^{k-1}\right) = p\left(q_k|q^{k-1}\right)p\left(q^{k-1}|Z^{k-1}\right). \tag{20}$$

Next, the update is given by:

$$p\left(q^k|Z^k\right) = \frac{p\left(Z_k|Z^{k-1},q^k\right)p\left(q^k|Z^{k-1}\right)}{p\left(Z_k|Z^{k-1}\right)}. \tag{21}$$

In practice, we do not separate the recursion into the prediction and update steps (20-21) and use the combined form (19) directly. Hence, we need not consider global hypotheses $q^k$ whose cardinality assumptions are inconsistent with the data cardinality $|Z_k|$. However, this does not constitute an implicit use of $|Z_k|$ in the prediction equation (20). Rather, in using (19) directly, we avoid considering those global hypotheses that are inconsistent with $|Z_k|$, as these will have null posterior probability.

A further concern raised by Mahler in [1, pp. 340-341] is that the labelling of measurements introduces an *a priori* order, and this in turn constitutes extraneous information that may introduce a statistical bias in the MHT solution. In fact, the labelling of measurements is arbitrary, it does not introduce an ordering, and it does not impact the computation of $p\left(q^k|Z^k\right)$. Hence, the labeling of measurements has no impact on the resulting MHT solution $\left(\hat{X}^k, \hat{q}^k\right)$ given by (2-3).

## 2.0 MULTI-INT SURVEILLANCE

MHT-based approaches are severely challenged when faced with highly disparate multi-INT data whereby high-revisit rate kinematic sensor data is to be fused with sporadic but highly-informative identity information. An illustration of the multi-INT challenge is in Figure 2-1. The challenge is to associate high-purity kinematic tracks with infrequently-arriving identity information, in settings where object density is high and many association possibilities must be reasoned over.

Distributed MHT [14-15] provides some computational savings. In a first MHT processing stage, we form short-duration high-purity kinematic tracks, followed by a fusion of track-level kinematic data and identity tracks in a second MHT processing stage. While this approach improves upon centralized MHT processing, it too does not scale to large, high-density scenarios with highly infrequent identity information. Next, we discuss two approaches to improving the state-of-the-art in association-based MTT for this problem.

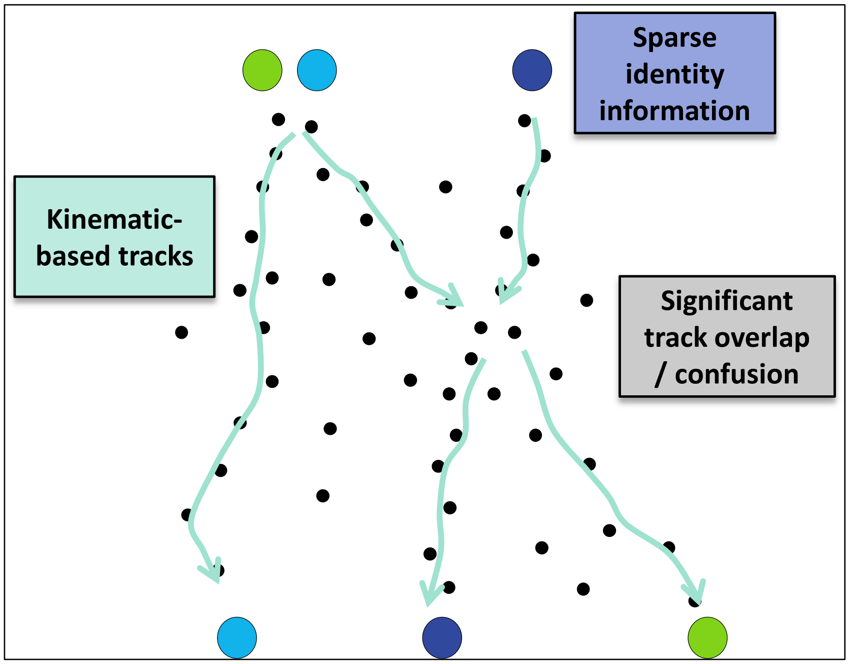


**Figure 2-1: Illustration of the Multi-INT Challenge.**

## 2.1 Asynchronous MHT

While MHT processing is effective for kinematic tracking, its application for our multi-INT processing is extremely challenging due to the need for deep hypothesis trees to benefit from highly-informative target emissions. Consider the following illustrative example.

Assume there are $N$ sensor scans, where the first and last scans are due to the low-rate sensor and intervening scans are due to the high-rate sensor. We assume one-dimensional MOU target motion and positional sensor measurements. We consider a number of solution schemes. The first is the *clairvoyant* solution, where measurement provenance is assumed to be known for the high-rate sensor as well. This reduces to a set of linear filtering problems for which the KF provides an optimal solution.

The second solution is to use the *global nearest neighbor* (GNN) assignment with sequential processing of all sensor scans. Note that, in general, data association errors do occur. We recover from such errors at the last scan (from the low-rate sensor), when measurement provenance is known. Naturally, for a fixed number of targets and target density, as the number of scans of data increases, the problem becomes more difficult in the sense that data association errors will accrue prior to the last scan of data.

Can we do better than the GNN solution if we constrain ourselves to maintaining a single global hypothesis? It turns out that improved performance is possible. This is achieved by performing Kalman smoothing based on the current state estimates at time $t_k$ and the final scan of measurements at time $t_N$, to estimate target positions at time $t_k$. These estimated positions can be used in defining the GNN assignment matrix, resulting in a more reliable solution than is possible with sequential processing. We call this approach *asynchronous GNN.*

Figure 2-2 illustrates one realization of target trajectories, along with four candidate solutions. These are the clairvoyant solution, the sequential GNN solution, and two variations on the asynchronous GNN solution – one with scoring based on approximate Kalman smoothing, and one exact scoring based on Kalman filtering. Note the "recovery" at the last scan exhibited by the sequential GNN solution. Figure 2-3 illustrates Monte Carlo performance results as a function of the number of sensor scans. When there are only two scans of data, both from the low-rate sensor, all four solutions coincide. The clairvoyant solution improves slightly with an increasing number of scans, due to filter convergence. The three solutions for which measurement provenance on high-rate sensor returns is unavailable all degrade with increasing number of scans, measured in terms of average position estimation error.

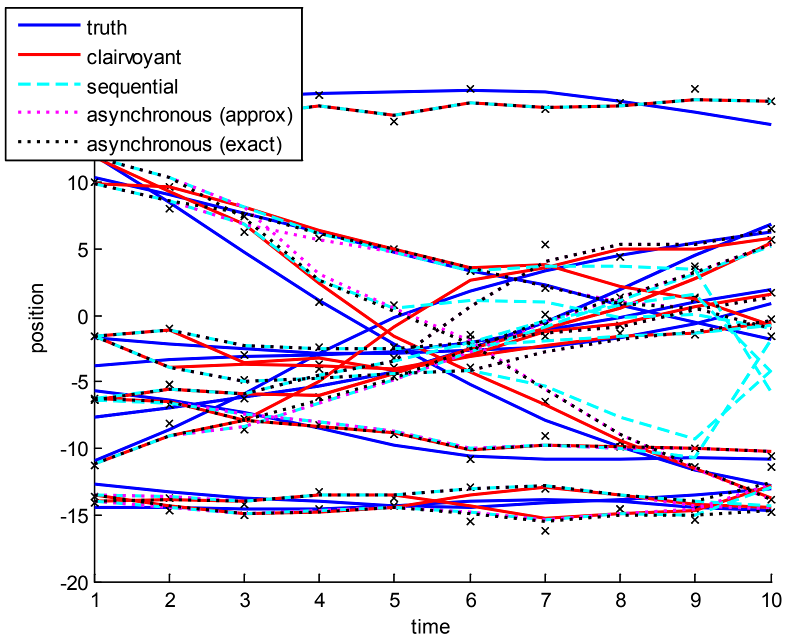


**Figure 2-2: Realization of Competing Solutions for Multi-Target Filtering.**

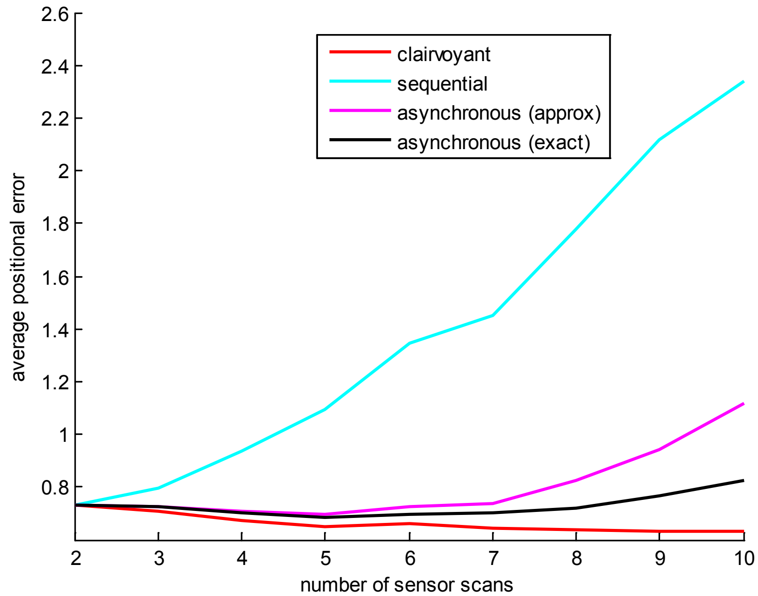


**Figure 2-3: Performance as a Function of Scenario Duration.**

We see that the asynchronous GNN provides a dramatic multi-target filtering improvement over sequential GNN, while maintaining the same processing complexity, albeit with the need for Kalman smoothing or an additional Kalman filtering update in defining the GNN assignment matrices. Note that in the asynchronous GNN solutions the information in the final scan is used solely to improve association decisions, and does not impact filter updates. There is no issue of repeated use of final-scan information.

The above result may be applied to the general MTT problem, for which the number of targets is unknown, as well as in MHT processing, where data association decisions are based on sliding window of scans and multiple association hypotheses are maintained. This is best described via a notional example.

Figure 2-4 illustrates *Asynchronous MHT* (A-MHT) processing of track level identity data S1 and kinematic data W1 and W2. The processing proceeds in batch or forensic mode. We initialize the set of track hypothesis trees with all unassociated identity tracks. Next, we proceed to process kinematic tracks sequentially, whereby the entire track is processed in forming and scoring track hypothesis trees; this in analogous to the preceding asynchronous GNN discussion. However, here we consider as well new target hypotheses. Further, we consider several processing steps before pruning the set of track hypothesis trees, as prescribed under MHT *n-scan* pruning logic.

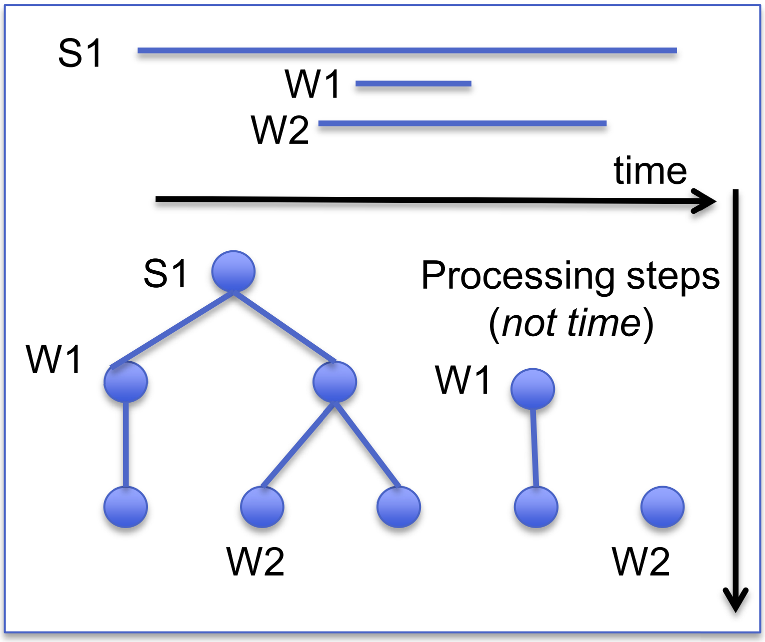


**Figure 2-4: A-MHT Hypothesis Formation via Batch Processing of Tracks.**

The track order for processing is somewhat arbitrary, but for convenience we order kinematic tracks by time, starting with the first track to terminate. This explains why, in the example, we process kinematic track W1 first. We do not consider the hypothesis that both W1 and W2 are due to the same target as they overlap in time. This would require a redundant-measurement sensor model, for which recent developments in MHT are discussed in [26].

A-MHT hypothesis generation logic is different from classical MHT in certain details as well. As an example, there is no need to consider limited track coasting prior to a track termination hypothesis. The A-MHT will allow for arbitrarily long track coasts, since the single-sensor tracks in any fused track hypothesis may exhibit significant temporal separation. Further details on our A-MHT advances may be found in [16, 28].

## 2.2 Multi-INT Graph-Based Tracking

An approach to track stitching that avoids hypothesis explosion relies on a Markovian assumption that simplifies likelihood computations, where $y_i$ represents a track (a sequence of associated measurements) and $y^i = (y_1, \dots, y_i)$:

$$L(y^n) = L(y_1) \prod_{i=2,\dots,n} L\left(y_i | y^{i-1}\right) \approx L(y_1) \prod_{i=2,\dots,n} L(y_i | y_{i-1}). \tag{22}$$

This simplification is generally valid for kinematic data, but is not so for identity data whose value does not degrade over time. The approximation assumes temporally non-overlapping tracks. The approach has been exploited fruitfully in kinematic sensor large-scale tracking application via a min-cost network flow formulation [29-30]. Unfortunately, the *graph-based tracking* (GBT) methodology is not directly applicable to the multi-INT challenge of interest here. Extensions to the single-target identity case may be found in [31-32].

It is of interest to exploit the computational reduction available in GBT while seeking to extend the formulation to the multi-INT setting. This generalization yields the *multi-INT GBT* (MI-GBT). While developed independently, the MI-GBT may be seen as a generalization to recently-reported video tracking based on multi-commodity flow ideas [33]. Indeed, this earlier work does not contend with multi-sensor data association.

We consider a set of kinematic tracks that we represent by a set of nodes $V$. We consider as well source and sink nodes, denoted by $v_0$ and $v_\infty$, respectively. We define the augmented set of nodes by $\bar{V} = V \cup \{v_0, v_\infty\}$. We consider a directed graph $G = (\bar{V}, A)$, where $A$ is a set of edges. For each feasible edge $(i,j) \in A$, we have a corresponding cost $c_{ij}$, given by a negative log likelihood, $c_{ij} = -logL\left(v_j|v_i\right)$.

An edge cost accounts for both detection and kinematic information; this includes accounting for the lack of kinematic detection between the end of $v_i$ and the start of $v_j$. All kinematic track nodes $v_i \in V$ have edges from the source and to the sink node, with costs $c_{0i}$ and $c_{i\infty}$, respectively. These costs reflect statistics for target birth, death, and kinematic motion, and for sensor detection and localization. Consistent with the standard Bernoulli detection model, we do not consider edges between time-overlapping kinematic tracks.

The kinematic graph-based tracking problem amounts to the following ILP problem. Further, the problem can be relaxed to a *min-cost network flow* (MCNF) problem, for which efficient integer-solution algorithms exist. We seek the solution that minimizes the objective (23) subject to constraints (24-26). Eqns. (25-26) insure that all nodes are used exactly once, and that flow balance is achieved:

$$J = \sum_{(i,j)\in A} c_{ij} x_{ij}, \tag{23}$$

$$x_{ij} \in \{0,1\}, \forall (i,j) \in A, \tag{24}$$

$$\sum_{i:(i,j)\in A} x_{ij} = 1, \forall j \in V, \tag{25}$$

$$\sum_{j:(i,j)\in A} x_{ij} = 1, \forall i \in V. \tag{26}$$

For the multi-INT problem, in addition to the kinematic tracks identified by the set $V$, we are given a set of emitter tracks $E$. For each emitter track $e_k \in E$, we identify the set of feasible kinematic track nodes, denoted by $V_k \subset V$. Feasibility is based on intersection of forward and backward light cones from each emission in track $e_k$, based on a maximum target speed constraint. Correspondingly, we have the augmented set of set of nodes $\bar{V}_k = V_k \cup \{v_0, v_\infty\}$ and the set of feasible edges $A_k \subset A$ for emitter track $e_k$. We define as well the null emitter $e_0$ and the augmented emitter set $\bar{E} = E \cup \{e_0\}$. Note that $V_0 = V$ and $A_0 = A$.

We consider the following ILP formulation. Here, $x_{ijk}$ denotes the directed edge from node $v_i$ to $v_j$ in the sub-graph $G_k = (\bar{V}_k, A_k)$. We have $G_0 = G$. Note that in general the cost $c_{ijk}$ depends on the emitter $e_k$, for transitions from the source or to the sink node, i.e. for $i = 0$ or $j = \infty$. It is necessary to include the edge $(0,\infty) \in A_k$ for all $k$ such that $e_k \in E$; we do not include $(0,\infty)$ in $A_0$:

$$J = \sum_{e_k\in\bar{E}} \sum_{(i,j)\in A_k} c_{ijk} x_{ijk}, \tag{27}$$

$$x_{ijk} \in \{0,1\}, \forall (i,j) \in A_k, \forall k\ s.t.\ e_k \in \bar{E}, \tag{28}$$

$$\sum_{e_k\in\bar{E}} \sum_{i:(i,j)\in A_k} x_{ijk} = 1, \forall j \in V, \tag{29}$$

$$\sum_{v_i\in\bar{V}\setminus\{0\}} x_{0ik} = 1, \forall k\ s.t.\ e_k \in E, \tag{30}$$

$$\sum_{j:(j,i)\in A_k} x_{jik} - \sum_{j:(i,j)\in A_k} x_{ijk} = 0, \forall i\ s.t.\ v_i \in V, \forall k\ s.t.\ e_k \in \bar{E}. \tag{31}$$

We seek the solution that minimizes the objective (27) subject to constraints (28-31). Eqn. (29) insure that all nodes are used exactly once. Eqn. (30) insures that all emitter are used exactly once. Eqn. (31) insures that flow balance is achieved in each sub-graph.

Consider the multi-INT data illustrated in Figure 2-5. There is a single emitter that emits twice. There are three kinematic tracks. Based on time-space feasibility, kinematic track $v_3$ and emitter track $e_1$ cannot originate from the same target. For this problem, the graphs $G_0$ and $G_1$ are illustrated in Figure 2-6.

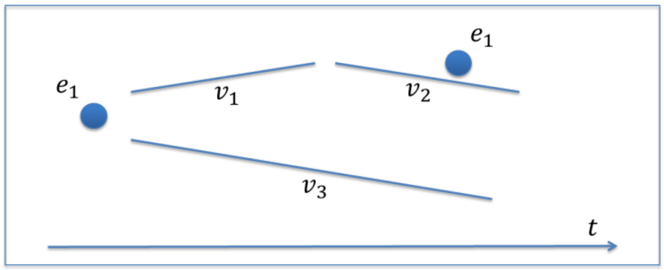


**Figure 2-5: A Small Example to Illustrate the Proposed Multi-INT Solution.**

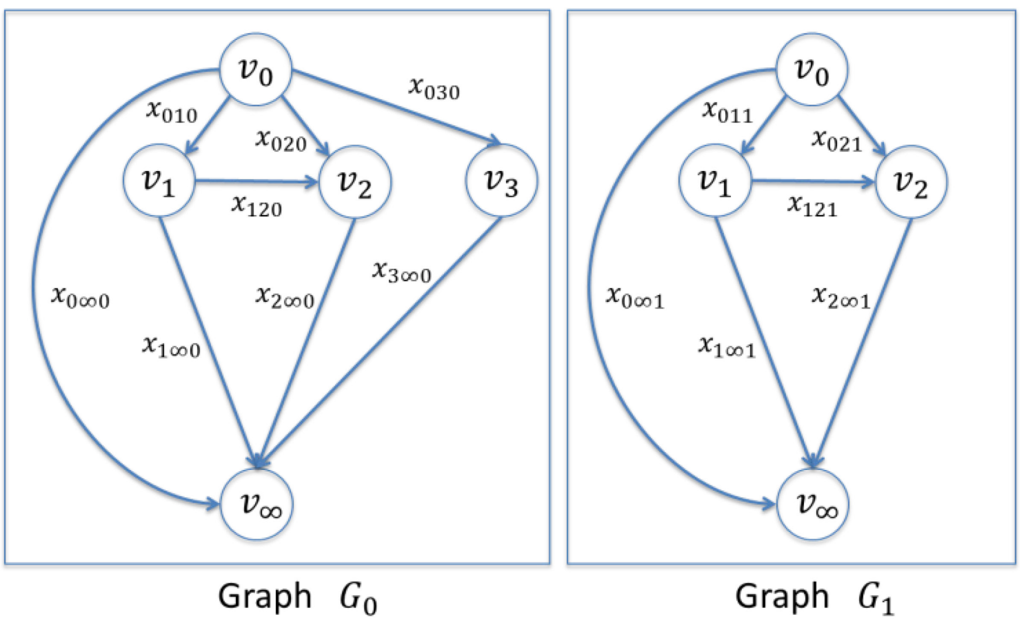


**Figure 2-6: Representation of the Multi-INT Graph Structure.**

Notionally, for this problem, the optimal solution that minimizes $J$ in eqn. (6) will be the following: $x_{030} = x_{3\infty0} = x_{011} = x_{121} = x_{2\infty1} = 1$, with all other variables $x_{ijk}$ equal to zero.

Note that the number of variables to be determined in solving the *integer linear program* (ILP) is $O(|\bar{V}|^2)$ for the single-INT problem, and $O(|\bar{V}|^2|\bar{E}|)$ for the multi-INT problem. While the multi-INT problem is larger, in general it is *much* smaller than the ILP size for an MHT-based formulation.

It is useful to have a back-of-the envelope assessment of the computational complexity associated with MHT, GBT, and MI-GBT solutions to the multi-INT problem. In particular, we wish to estimate the size of the ILP associated with these paradigms. Given $m$ sets of $|V|$ kinematic tracklets and $|E|$ emitter tracks, the GBT problem size is $M = O(m|V|^2)$ while the MI-GBT problem size is $M = O\left(m|V|^2(1 + |E|)\right)$. Both compare favorably to MHT-based approaches (including the A-MHT), for which problem size is $M = O\left(|V|^{m+1}(1 + |E|)\right)$. The solution time associated with the ILP is problem-size dependent. We assume $O(M^4)$ for A-MHT and MI-GBT based on LP relaxation, and $O(M^3)$ for the GBT based on *min-cost network flow* (MCNF) or an equivalent bipartite matching formulation.

For nominal choices of the problem-size parameters, estimated execution time as a function of scenario duration (measured as the number of sets of kinematic tracklets $m$) are illustrated in Figure 2-7. Determining the exact solution via MHT processing incurs exponentially-increasing complexity. The MI-GBT provides an approximate solution with somewhat higher complexity as the GBT that fails to exploit identity constraints.

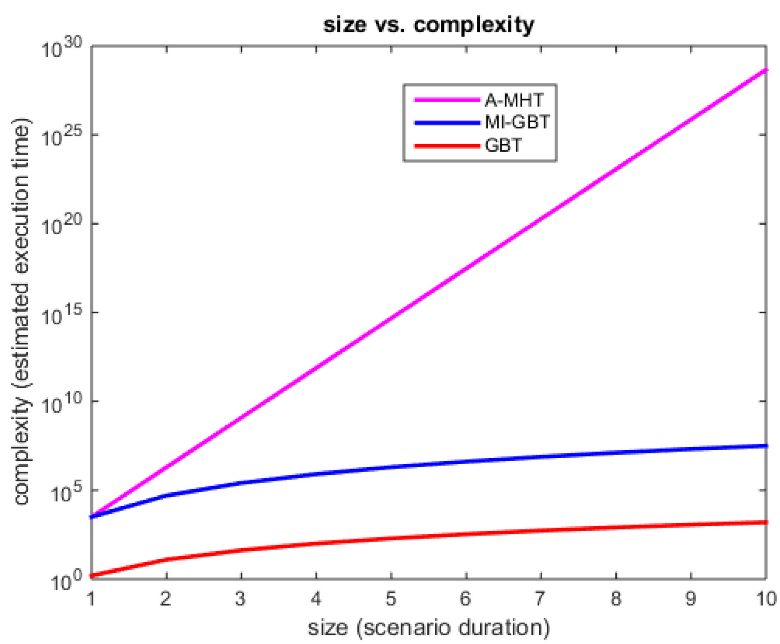


**Figure 2-7: MI-GBT Achieves Near-Optimal Performance at Dramatically Reduced Complexity with Respect to A-MHT. The GBT approach cannot account for multi-INT constraints.**

The network-flow formulation that can be used to solve the GBT problem ensures that integer solutions are obtained. We do not have this guarantee when solving an ILP with a more general LP relaxation approach, as required in the MI-GBT. Nonetheless, non-integer solutions are seldom encountered in practice. When they are, a simple solution round-off scheme (while maintaining feasibility) can be performed. Non-integer solutions may contain information that could be exploited in a more involved solution round-off methodology. Further details and simulation results for the MI-GBT may be found in [34-35].

## 3.0 CONCLUSIONS

MHT is a leading paradigm for advanced MTT. In this manuscript, we discuss salient elements of the mathematical underpinning of MHT. We seek to address some of the concerns that have been raised in the research community, including the use of a MAP optimality criterion and of the machinery of Bayesian inference.

Additionally, we discuss recent extensions to address the multi-INT fusion problem (the correlation of highly disparate multi-sensor data), which is a significant challenge for the MTT research community. The MHT paradigm has been enhanced with the development of the A-MHT that exploits the forensic nature of the multi-INT problem, achieving improved data association via out-of-sequence processing.

Inspired by research in the video-tracking community, graph-based methods have fruitfully been applied to broader classes of surveillance problems. The GBT provides good computationally performance, subject to a mild path-independence assumption that is valid in many single-sensor surveillance settings. Unfortunately, extensions to address the multi-INT challenge have proved elusive.

The MI-GBT is a multi-INT generalization to the GBT that introduces further mild simplifying assumptions so as to enable a pairwise-cost formulation of the multi-INT problem. This leads to an ILP optimization problem that avoids the problematic (nonlinear) identity constraints that cannot be handled in the GBT formalism, and is much smaller than the ILP associated with MHT-based methods.

Current directions for future research in A-MHT and MI-GBT include algorithmic extensions to account for (i) evasive *move-stop-move* target motion with motion-sensitive kinematic sensors, and (ii) the redundant-measurement phenomena observed in real surveillance data.

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